NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL MEMORANDUM

No. 1179

DETERMINATION OF THE STRESS CONCENTRATION FACTOR

OF A STEPPED SHAFT STRESSED IN TORSION BY

MEANS OF PRECISION STRAIN GAGES

By A. Weigand

TRANSLATION

"Ermittlung der Formziffer der auf Verdrehung beanspruchten
abgesetzten Welle mit Hilfe von Feindehnungsmessungen."
Luftfahrt-Forschung Band 20, Lieferung 7,
pp. 217-219, München, July 20, 1943

Washington

September 1947

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL MEMORANDUM NO. 1179

DETERMINATION OF THE STRESS CONCENTRATION FACTOR

OF A STEPPED SHAFT STRESSED IN TORSION BY

MEANS OF PRECISION STRAIN GAGES*

By A. Weigand

The stress distribution in stepped shafts stressed in torsion is
determined by means of the electric precision strain gage by Lehr
and Granacher [5]; the stress concentration factor α_k as a function
of $\frac{d}{D}$ and $\frac{\rho}{D}$ for $0.5 \leqq \frac{d}{D} \leqq 0.9$ and $0.1 \leqq \frac{\rho}{d} < 0.25$ is ascertained
from the measurements. It is shown that the test values always are
slightly lower than the values resulting from an approximate formula
by Sonntag [2].

Outline: I. Test Setup and Measuring Procedure

II. Evaluation of the Measurements and Results

III. Summary

IV. References

The stress distribution in stepped shafts stressed in torsion
was first determined by F. A. Willers [1] by approximate integration
of the differential equation of the stress function. R. Sonntag [2]
gave an approximate formula for the stress function and the maximum
stress for the case $D - d \geqq 2\rho$. (See fig. 1.) L. S. Jacobsen,
A. Thum, and W. Bautz ([3] and the bibliography quoted there)
ascertained the concentration factor by means of an electric model.
The values found in this manner do not always agree with those calcu-
lated according to Sonntag; a discussion between these scientists
ensued [4].

In order to definitely clear up this problem the stress distri-
bution of steel shafts was determined by means of the precision
strain gage developed by E. Lehr and H. Granacher [5].

I. TEST SETUP AND PROCEDURE

Figure 2 shows the apparatus which was used for twisting of the
test shaft. Its modus operandi is explained in figure 3. The test
shaft is obviously stressed by a pure torque. This fact was confirmed
by a brittle lacquer test; the principal stress curves had an
inclination of 45° toward the generatrices of the cylindric shaft.

The test points 1 to 11 for the strain measurements were
arranged according to figure 5 on three generatrices I, II, and III
which were 90° to each other. The two principal strains ϵ_1 and ϵ_2,
which must be equal and opposite for pure torsion, were measured.
Actually,the values deviate from the average usually by 3 to 4 percent;
the reason is directional inaccuracy of the test section which was
marked with a prick punch (1.3 mm) and a slight bending stress of the
shaft caused by the fact that the twisting forces designated by α
in figure 3 are not exactly equal. This error is essentially elimi-
nated by measuring on three generatrices, figure 5. From the three
test values which lie on the same circle the mean value is taken.

The strains were measured by means of the precision strain
gage of 1.3-millimeter gage length developed by E. Lehr and
H. Granacher; its modus operandi is described in detail in [5].
Before starting and at the end of each test series,which lasted three
to four days per shaft,the instrument was calibrated on a tensile bar
of known modulus of elasticity; the values found never differed by
more than 1 to 2 percent and lay within the errors of measurement.
It is true, the filament current for the small bulb of the strain
gage must be kept exactly constant; at an intensity of the filament
current of 120 mA fluctuations of 0.1 mA are already troublesome.
Loose contacts, in particular, must be carefully avoided since
irregular fluctuations would result in the multiflexgalvanometer
measuring the photocurrent which has a sensitivity of about 4×10^{-6}mA
per scale division.

For the tests an initial load of $P_O = 50$ kilograms was selected
and the deflection of the multiflexgalvanometer measuring the photo
current in dependence on the loading was measured. This dependence is
linear within the errors of measurement as can be seen in figure 6.
The galvanometer delection was referred to 400-kilogram load increment;
in figure 6 on the average 53 scale sectors correspond to this load

increment. Based on the (straight lined) calibration curve one now
knows what strain corresponds to this load increment.

II. EVALUATION OF THE MEASUREMENTS AND RESULTS

From the known relation

$$\left.\begin{array}{l}\sigma_1 = \dfrac{E}{1 - \mu^2}\left(\epsilon_1 + \mu\epsilon_2\right) \\[2em] \sigma_2 = \dfrac{E}{1 - \mu^2}\left(\epsilon_2 + \mu\epsilon_1\right)\end{array}\right\} \qquad (1)$$

between the principal strains ϵ_1, ϵ_2 and the principal stresses
σ_1, σ_2 for the plane stress condition one obtains for the shear stress
τ in the cross section of the shaft

$$\tau = \frac{\sigma_1 - \sigma_2}{2} = \frac{E}{1 + \mu}\,\frac{\epsilon_1 - \epsilon_2}{2} \qquad (2)$$

The moduli of elasticity and of shear of the test shafts which were
manufactured of St. C 45. 61., resulted as $E = 2.14 \times 10^6$,
$G = 0.82 \times 10^6$ kg cm^{-2} and thus $\mu = \frac{1}{2}\frac{E}{G} - 1 = 0.305 \approx 0.30$.
Thus τ is for the present case

$$\tau = 0.823\left(\epsilon_1 - \epsilon_2\right) \times 10^6$$

In figure 7 the distribution of shear stress along the shaft is
represented for the case $\rho/d = 0.107$, $d/D = 0.70$. One can see that
for the smooth part of the shaft the shear stress is constant except
for slight scatter. The fillet begins at test point 8 shortly
before the shear stress starts to increase and reaches its maximum at
about test point 9 which lies already within the fillet. The resulting
stress concentration factor for the present case is:

$$\alpha_k = \frac{\tau_{max}}{\tau_o} = \frac{715}{520} = 1.38$$

In the same manner the stress concentration factor was determined for
a series of values of $\rho/d = \alpha$ and $d/D = \beta$. Table 1 shows the result.
Fairing these test values graphically, one obtains the figures
designated in table 1 as faired test values. These values are

plotted in figures 7 and 9 as functions of d/D and ρ/d. They are
always somewhat lower than the values calculated from the approximate
formula by Sonntag

$$\alpha_k = \beta(1.5 + 3.0\alpha)\ \frac{1 + 4\alpha}{1 + 6\alpha} + (1 - \beta - 2\alpha\beta)\ \left(1 + \frac{1}{12\alpha}\right) \quad (3)$$

which is valid only for $\beta \lesseqgtr \dfrac{1}{1 + 2\alpha}$. Taking the unavoidable errors
of measurement into consideration (for instance, inaccuracy of the
punch mark, not ideal torsion) it will be permissible to assume the
accuracy of these faired values with about 5 percent.

III. SUMMARY

The stress distribution in stepped shafts stressed in torsion
was determined by means of percision strain gages. Comparison with
an approximate formula set up by R. Sonntag showed that according to
that formula the stress concentration factor α_k in the interval
$0.1 \lesseqgtr \dfrac{\rho}{d} \lesseqgtr 0.25$ and $0.5 \lesseqgtr \dfrac{d}{D} \lesseqgtr 0.9$ can be calculated with sufficient
accuracy.

Translated by Mary L. Mahler
National Advisory Committee
for Aeronautics .

IV. REFERENCES

1. Willers, F. A.: Die Torsion eines Rotationskörpers um seine
        Achse. Z. Math. u. Phys. Bd. 55 (1907) p. 225.

2. Sonntag, R.: Zur Torsion von runden Wellen mit veränderlichem
        Durchmesser. Z. angew. Math. u. Mech. Bd. 9 (1929) p. 1.

3. Thum, A. and Bautz: Zur Frage der Formziffer. Z. VDI Bd. 79
        (1935) p. 1303.

4. Sonntag, R., Thum, A., and Bautz, W.: Zur Frage der Formziffer.
        Z. VDI Bd. 81 (1937) p. 561.

5. Lehr, E., and Granacher, H.: Dehnungsmessgerät mit sehr kleiner
        Messtrecke und Anzeige mittels Sperrschicht-Photozelle.
        Forsch. a. d. Geb. d. Ingenieurwesens Bd. 7 (1936) p. 66.

TABLE I

THE STRESS CONCENTRATION FACTOR α_k AS A

FUNCTION OF ρ/d AND d/D

ρ/d	d/D	α_k Measured	α_k Faired	α_k Calculated
0.25	0.50	1.17	1.22	1.23
	.667	1.18	1.19	1.20
	.824	1.21	1.13	----
	.90	1.13	1.09	----
.214	.50	1.24	1.25	1.27
	.70	1.22	1.20	1.22
	.824	1.19	1.19	----
	.90	1.13	1.10	----
.15	.50	1.33	1.36	1.37
	.70	1.22	1.28	1.30
	.80	1.20	1.23	----
	.90	1.13	1.15	----
.107	.50	1.43	1.45	1.49
	.60	1.43	1.41	1.44
	.70	1.38	1.35	1.38
	.80	1.26	1.30	1.32
	.90	1.19	1.22	----
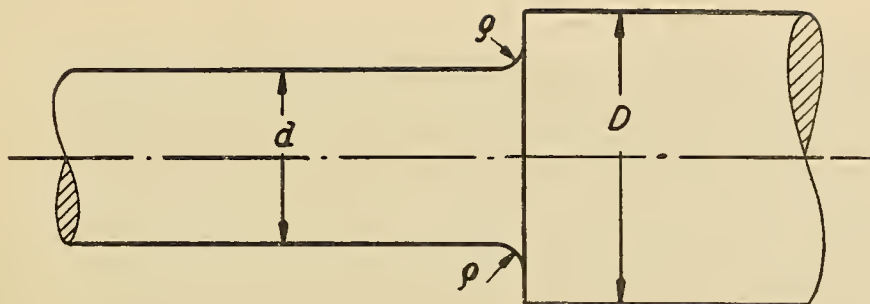
Figure 1.- Designations at the stepped shaft.

Figure 2.- Torsion machine.
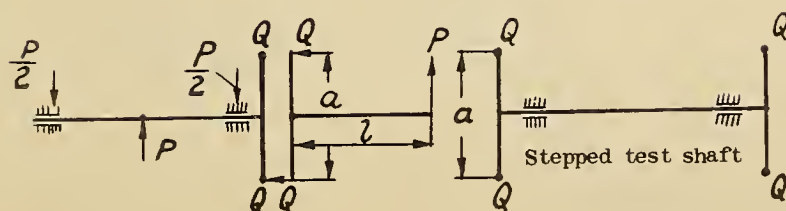
Figure 3.-   Sketch explaining the operation method of the torsion machine.
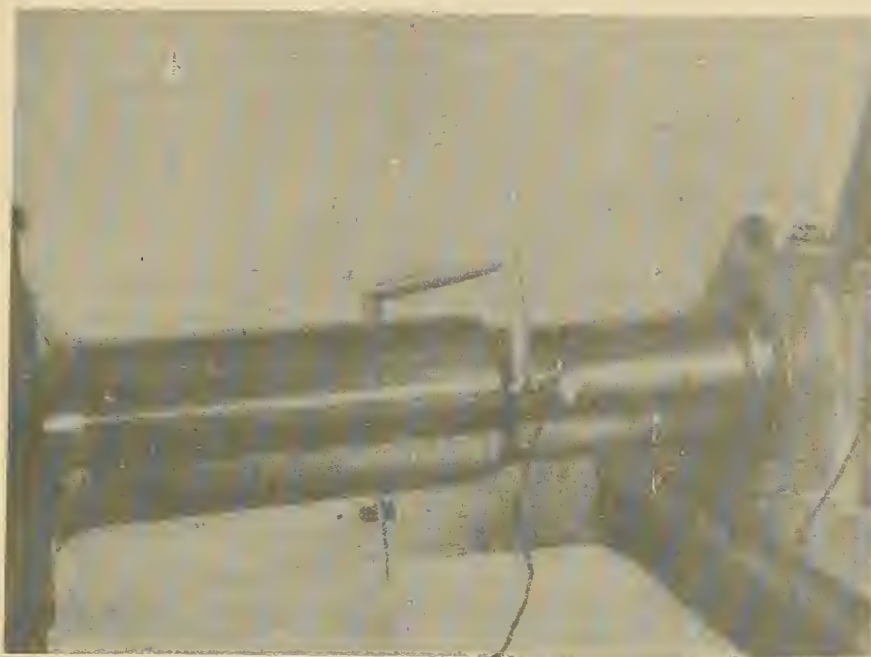
Figure 4.- Test shaft with strain gauge.
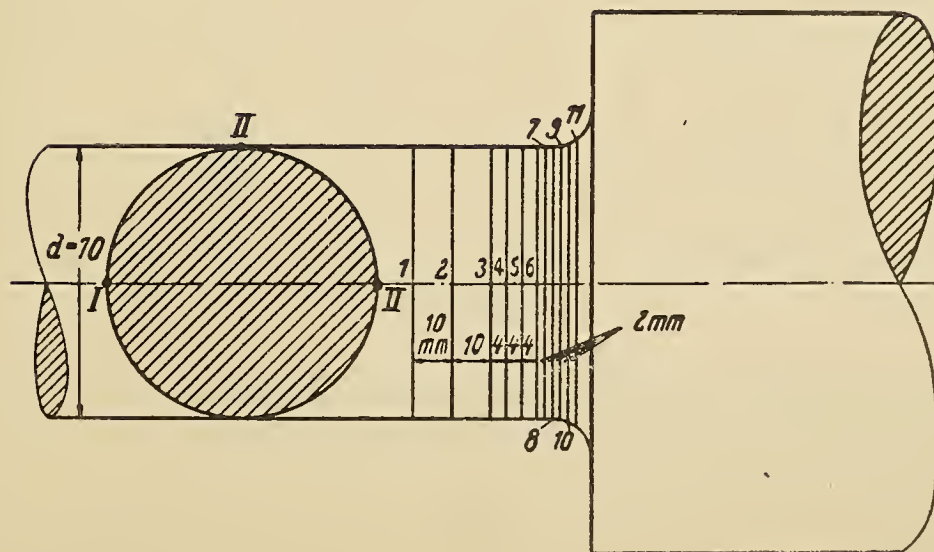

Figure 5.- Arrangement of the test points 1 to 11 on the generatrices
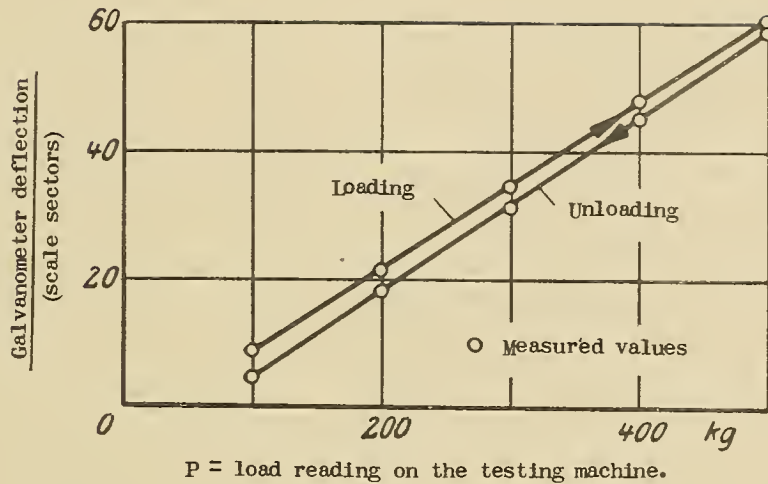I to III of the test shaft.

Figure 6.- Dependence of the photo current (galvanometer - deflection) on the load reading of the testing machine for a certain test point.
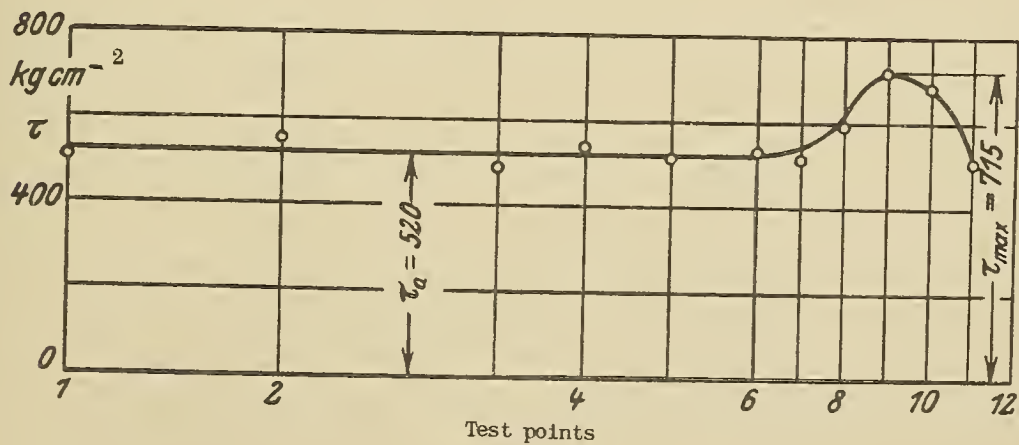

Figure 7.- Distribution of the shear stress along a generatrix of the stepped shaft for $\rho/d = 0.107$, $d/D = 0.70$.
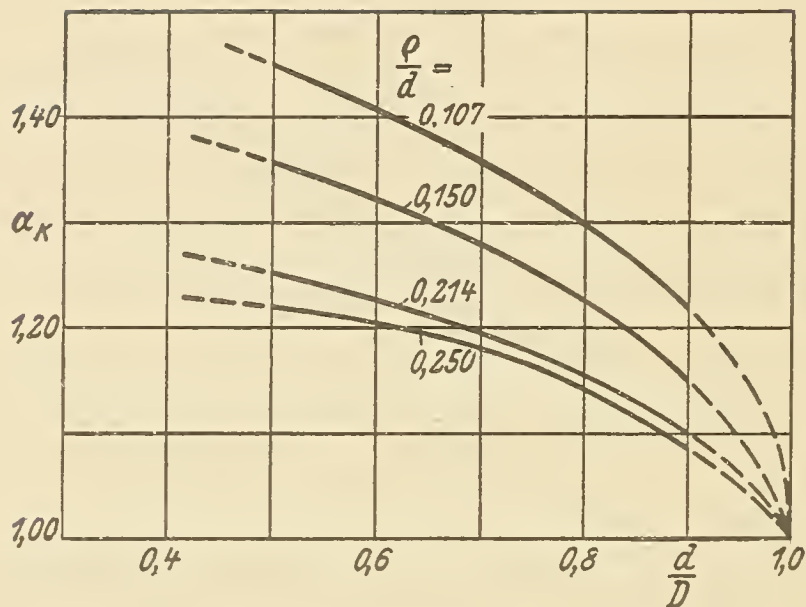
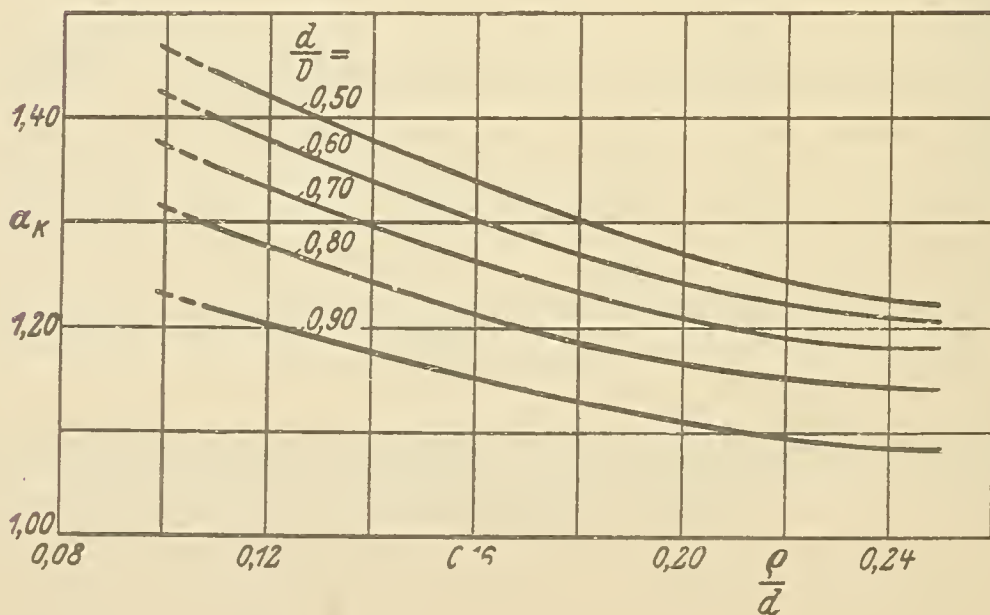Figure 8.- The concentration factor α_k as a function of d/D.

Figure 9.- The concentration factor α_k as a function of ρ/d.